Sarvajani Education Society
Sir P. T. Sarvajanik College of Science, Surat
Department of Mathematics
MATHEMATICS CLUB

Mαthematics Newsletter

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Ambimagic squares

A shortcoming of the usual $N \times N$ magic square, it could be said, is an asymmetry in its very definition: all $2N$ orthogonals must yield the same sum, while only two of the $2N$ diagonals need comply. This is remedied in panmagic squares, but another solution is found in what I call *ambimagic* or *a*dditive-*m*ultiplicative *bi*-magic squares, in which again all $2N$ orthogonals must yield the same sum, while all 2N diagonals are to yield the same *product*. This *4×4* example uses the smallest possible distinct positive integers:

$$\begin{matrix} 1 & 33 & 4 & 22 \\ 28 & 10 & 7 & 15 \\ 11 & 3 & 44 & 2 \\ 20 & 14 & 5 & 21 \end{matrix}$$

$S = orthogonal\ sum = 60$,
$P = diagonal\ product = 9240$.

Note that the product of the 4 numbers in every toroidally-connected 2×2 subquare is also P. On tiling the plane with this square, every 4×4 area will be ambimagic. The following 4×4 square uses the set of smallest possible distinct integers, half of them negative:

$$\begin{matrix} -6 & -5 & 10 & 1 \\ -8 & -3 & 9 & 2 \\ 6 & 5 & -10 & -1 \\ 8 & 3 & -9 & -2 \end{matrix} \qquad S = 0, P = 360$$

4×4 *ambimagics* may simultaneously be ordinary magic squares:

$$\begin{matrix} -3 & 1 & -1 & 3 \\ -15 & 5 & -5 & 15 \\ 12 & -4 & 4 & -12 \\ 6 & -2 & 2 & -6 \end{matrix} \qquad S = 0, P = 360$$

The two main diagonals add to zero, as do the rows and columns.

A general formula for order 3 is:

$$\begin{matrix} a & -(a+b) & b \\ -a(c+1) & (a+b)(c+1) & -b(c+1) \\ ca & -c(a+b) & cb \end{matrix}$$

showing that in 3×3 squares some entries are always $-ve$ and S is always zero, while $P = abc(a+b)(c+1)$. That specimen using the smallest integers is:

$$\begin{matrix} 1 & -3 & 2 \\ -4 & 12 & -8 \\ 3 & -9 & 6 \end{matrix} \quad S = 0, P = 72$$

Note that in the toroidally-connected square:

1. Every entry is the sum of its four neighbouring corner entries,

   e.g. $12 = 1 + 2 + 3 + 6$, or

   $-4 = -3 + -9 + 6 + 2$.

2. The product of the two diagonal entries of any 2×2 subsquare equals the product of the two other diagonal entries;

   e.g. $1 \times 12 = -3 \times -4$, or

   $1 \times 6 = 2 \times 3$.

3. The sum of any two entries in the same orthogonal equals the sum of the two entries not in any orthogonal common to either;

   e.g. $-3 + 12 = 3 + 6$, or

   $-4 + 3 = -3 + 2$.

4. Just like the classes of orthogonally (additive) magic, or diagonally (multiplicative) magic squares of which the class of ambimagics is the intersection, permuting rows and/or columns preserves their properties, which means that on tiling the plane with such a square we can then outline any 3×3 area to discover an ambimagic square.

Using point (2) above, it is easy to see that every entry in a 3×3 ambimagic square is equal to the product of its 4 orthogonal neighbours, divided by P. Hence, given any numerical square, and then dividing every entry by the cube root of P, yields a new square for which $P = 1$. Trouble is, many of the entries may then be irrational numbers. To avoid this, one strategy is start off with an ambimagic square using integers, and for which P is a perfect cube. Dividing all nine integers by the cube-root of P then results in rational entries, and with $P = 1$. Every entry will then be equal to the product of its 4 orthogonal neighbours, whereas by (1) above, we know that every entry is also equal to the sum of its 4 diagonal neighbours. Below, one such square tiles the plane to result in what I call a magic carpet:

$$
\begin{array}{cccccccc}
 & & & \vdots & & & & \\
 & \frac{1}{15} & -\frac{9}{10} & \frac{5}{6} & \frac{1}{15} & -\frac{9}{10} & \frac{5}{6} & \\
 & -\frac{1}{3} & \frac{9}{2} & -\frac{25}{6} & -\frac{1}{3} & \frac{9}{2} & -\frac{25}{6} & \\
\cdots & \frac{4}{15} & -\frac{18}{5} & \frac{10}{3} & \frac{4}{15} & -\frac{18}{5} & \frac{10}{3} & \cdots \\
 & \frac{1}{15} & -\frac{9}{10} & \frac{5}{6} & \frac{1}{15} & -\frac{9}{10} & \frac{5}{6} & \\
 & -\frac{1}{3} & \frac{9}{2} & -\frac{25}{6} & -\frac{1}{3} & \frac{9}{2} & -\frac{25}{6} & \\
 & \frac{4}{15} & -\frac{18}{5} & \frac{10}{3} & \frac{4}{15} & -\frac{18}{5} & \frac{10}{3} & \\
 & & & \vdots & & & &
\end{array}
$$

Any 3 × 3 area on this infinite carpet is an ambimagic square, in addition to which, every number is equal to both the sum of its 4 diagonal neightbours as well as the product of its 4 diagonal neighbours.

Mabimagic squares:

The dual of an ambimagic square is a "mabimagic" square (mabi to rhyme with "barbie" not "maybe"), being a square in which the orthogonals and diagonals switch roles. In the case of odd order squares the general formula is then merely a rearrangement of the ambimagic formula. Hence for order 3, as follows:

$$
\begin{array}{ccc}
-c(b+1) & ca & -(a+b) \\
a & (a+b)(c+1) & cb \\
-c(a+b) & b & -c(a+1)
\end{array}
$$

Applying this same transformation to the 3×3 ambimagic square shown above then yields:

$$
\begin{array}{rrr}
-8 & 3 & -3 \\
1 & 12 & 6 \\
-9 & 2 & -4
\end{array}
$$

in which the diagonals sum to 0 and othogonals multiply to 72.

This switchability does not apply to squares of even order. In odd order squares every orthogonal intersects with *one* other orthogonal and likewise for diagonals. But in even order squares each diagonal intersects with another diagonal *twice*. The symmetry between orthogonals and diagonals thus disappears.

The creation of even-ordered mabimagic squares poses interesting problems. Here is an especially beautiful 4×4 example. Replace A, B, C, D with any 4 distinct non-zero numbers you please. The resulting 16 entries are then all distinct, while every orthogonal multiplies to 1 and every diagonal sums to 0.

A	B	$1/A$	$1/B$
C	D	$1/C$	$1/D$
$-1/A$	$-1/B$	$-A$	$-B$
$-1/C$	$-1/D$	$-C$	$-D$

The complementary pairs x and $-X$ are distributed as in Dudeney graphic type I, while the complementary pairs X and $1/X$ are distributed as in type V. Cyclic permutations of rows and/or columns do not change these properties. It is easy to prove that no ambimagic can be made using the same entries as this mabimagic square. For in that case, every row and column must sum to zero, so that $-A$ must appear in the same row and column as A. But $-A$ can occur but once.

A rather nice set of numbers to substitute for the variables here is $A = X^1$, $B = X^3$, $C = X^{27}$, and $D = X^{23}$, where X is a 32^{nd} root of unity, or the complex number = $e^{i\pi}/16$. A check will show that the remaining entries then correspond to the 16 odd numbered 32^{nd} roots of unity $(X^1, X^3, X^5, \ldots, X^{31})$, a family of points spaced evenly around the periphery of a unit circle centred on the origin of the Argand diagram. If it is symmetry that lends beauty to magic squares then nothing I have ever met excels this specimen.

The above square is an instance of a more general case:

$-a$	b	$-c$	$-f * d/b$
$-d$	$-e$	f	$-a * c/e$
c	$f * d/b$	a	$-b$
$-f$	$a * c/e$	d	e

$$p = -(a \times c \times d \times f)$$
$$S = 0$$

Note that the two main diagonals become multiplicatively magic when

$$(-a^2) \times (-e^2) = (-f^2) \times \left(-\left(f \times \frac{d}{b}\right)^2\right) = -(a \times c \times d \times f),$$

which implies

$c = -e \times f/b$ and $d = a \times e \times b/f^2$.

In that case we have a multiplicatively magic square that is also mabimagic, as for example:

12	1	−6	−18
9	3	−2	−24
6	18	−12	−1
2	24	−9	−3

—Lee Sallows

[*Lee Cecil Fletcher Sallows (born April 30, 1944) is a British Electronics Engineer known for his contributions to recreational Mathematics. He is particularly noted as the inventor of Golygons, Self-enumerating sentences, and Geomagic squares.*]

Courtesy: **https://www.leesallows.com** ■

Calculation of $\frac{d^{\frac{1}{2}}}{dx^{\frac{1}{2}}}(x)$

Let the function

$$f(x) = x^m$$

$$\Rightarrow \frac{d}{dx}(x^m) = mx^{m-1}$$

In general,

$$\frac{d^n}{dx^n}(x^m) = \frac{m!}{(m-n)!} \cdot x^{m-n}; m > n$$

Now, we know that,

$n! = \Gamma(n+1)$ (Gamma function)

So, using the Gamma function, we get,

$$\boldsymbol{\frac{d^n}{dx^n}(x^m) = \frac{\Gamma(m+1)}{\Gamma(m-n+1)} \cdot x^{m-n}; m \geq 0}$$

Now, substitute $m = 1$ and $= \frac{1}{2}$, we obtain,

$$\frac{d^{\frac{1}{2}}}{dx^{\frac{1}{2}}}(x^1) = \frac{\Gamma(1+1)}{\Gamma\left(1-\frac{1}{2}+1\right)} \cdot x^{1-\frac{1}{2}}$$

$$\frac{d^{\frac{1}{2}}}{dx^{\frac{1}{2}}}(x) = \frac{\Gamma(2)}{\Gamma\left(\frac{3}{2}\right)} \cdot x^{\frac{1}{2}}$$

$$\frac{d^{\frac{1}{2}}}{dx^{\frac{1}{2}}}(x) = \frac{1}{\frac{\sqrt{\pi}}{2}} \cdot x^{\frac{1}{2}}$$

$$\boldsymbol{\frac{d^{\frac{1}{2}}}{dx^{\frac{1}{2}}}(x) = \frac{2x^{\frac{1}{2}}}{\sqrt{\pi}}}$$

$\therefore$ Half derivative of x is $\frac{2x^{\frac{1}{2}}}{\sqrt{\pi}}$.

Note that:

1. $\Gamma(n+1) = n\Gamma(n)$
2. $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$

If we repeat the process on the result, we have,

$$\frac{d^{\frac{1}{2}}}{dx^{\frac{1}{2}}}\left(\frac{2x^{\frac{1}{2}}}{\sqrt{\pi}}\right) = \frac{2}{\sqrt{\pi}} \cdot \frac{\Gamma\left(\frac{1}{2}+1\right)}{\Gamma\left(\frac{1}{2}-\frac{1}{2}+1\right)} \cdot x^{\frac{1}{2}-\frac{1}{2}}$$

$$\therefore \frac{d^{\frac{1}{2}}}{dx^{\frac{1}{2}}}\left(\frac{d^{\frac{1}{2}}}{dx^{\frac{1}{2}}}(x)\right) = \frac{2}{\sqrt{\pi}} \cdot \frac{\Gamma\left(\frac{3}{2}\right)}{\Gamma(1)} \cdot x^0$$

$$\therefore \frac{d^{\frac{1}{2}}}{dx^{\frac{1}{2}}}\left(\frac{d^{\frac{1}{2}}}{dx^{\frac{1}{2}}}\right)(x) = \frac{2}{\sqrt{\pi}} \cdot \frac{\frac{\sqrt{\pi}}{2}}{1} \cdot 1$$

$$\therefore \frac{d}{dx}(x) = 1. \blacksquare$$

Probability of seeing a car on the road..!!!

If the probability of seeing a car on the road in 30 minutes is 0.95, what is the probability of seeing a car on the road in 10 minutes?

Let's consider every 10 minutes interval like a coin flip. Let p be the probability of not seeing a car in 10 minutes.

Let's find the probability of not seeing a car in 20 minutes. It's like repeating twice the event not seeing a car in 10 minutes.

$$\underset{(no\ cars\ in\ 20)}{P_b} = \underset{(no\ cars\ in\ 10)}{P_b} \times \underset{(no\ cars\ in\ 10)}{P_b}$$

$$\therefore \underset{(no\ cars\ in\ 20)}{P_b} = p(p)$$

$$\therefore \underset{(no\ cars\ in\ 20)}{P_b} = p^2$$

Now, let's find the probability of the event not seeing a car in 30 minutes. It's the probability of the event not seeing a car in 20 minutes, followed by the event not seeing a car in 10 minutes.

$$\underset{(no\ cars\ in\ 30)}{P_b} = \underset{(no\ cars\ in\ 20)}{P_b} \times \underset{(no\ cars\ in\ 10)}{P_b}$$

$$\therefore \underset{(no\ cars\ in\ 20)}{P_b} = p^2(p)$$

$$\therefore \underset{(no\ cars\ in\ 20)}{P_b} = p^3$$

we can directly calculate

$$\underset{(no\ cars\ in\ 30)}{P_b} = \underset{(no\ cars\ in\ 10)}{P_b} \times \underset{(no\ cars\ in\ 10)}{P_b} \times \underset{(no\ cars\ in\ 10)}{P_b} = p^3$$

Let's continue, the probability of seeing at least 1 car in 30 minutes is the complement event of seeing no cars in 30 minutes.

$$\underset{(car\ in\ 30)}{P_b} = 1 - \underset{(no\ cars\ in\ 30)}{P_b}$$

$$\therefore \underset{(car\ in\ 30)}{P_b} = 1 - p^3$$

This is equal to 0.95. We get an equation to solve for

$$1 - p^3 = 0.95$$

$$p^3 = 0.05$$

$$p = 0.05^{\frac{1}{3}} \approx 0.368$$

The probability of seeing at least 1 car in 10 minutes is the complement event of seeing no cars in 10 minutes.

$$\underset{(car\ in\ 10)}{P_b} = 1 - \underset{(no\ cars\ in\ 10)}{P_b}$$

$$\underset{(car\ in\ 10)}{P_b} = 1 - p$$

$$\underset{(car\ in\ 10)}{P_b} \approx 0.632$$

Finally, the answer is approximately 63 percent. ■

Mathematical Fallacies

Mathematical fallacies are errors, typically committed with an intent to deceive, that occur in a mathematical proof or argument. A fallacy in an argument doesn't necessarily mean that the conclusion is necessarily incorrect, only that the argument itself is wrong. However, fallacious arguments can have surprising conclusions, as shown:

Division by Zero

Let

$$\therefore x = 1$$

$$\therefore x^2 = x$$

$$\therefore x^2 - 1 = x - 1$$

$$\therefore \frac{x^2 - 1}{x - 1} = 1$$

$$\therefore \frac{(x-1)(x+1)}{x-1} = 1$$

$$\therefore x + 1 = 1$$

$$\therefore x = 0$$

But $x = 1$,

$$\therefore 1 = 0$$

The fallacy here is subtle. In step 2, multiplying both sides by x introduces an extraneous solution to the equation of $x = 0$. Then, in step 4, there is a division by $x - 1$, which is an illegal operation because $x - 1 = 0$ and you can't divide by zero. This illegal operation has the effect of leaving the extraneous solution $x = 0$ as the only solution to the equation.

You can use a similar method to "show" that any number is equal to any other number.

For example, to show that $7 = -4$:

Let

$$x = 7$$

$$\therefore x - 7 = 0$$

Multiply both sides by $x + 4$,

$$(x - 7)(x + 4) = 0$$

Divide both sides by $x - 7$

$$x + 4 = 0$$

$$\therefore x = -4$$

Infinite Series Fallacies

If you've taken mathematics in high school or first-year university or college, you may have learned that an infinite geometric series has a sum if it is convergent; that is, if the ratio between any term and the previous term is less than 1 and greater than −1. Geometric series that are not convergent cannot be summed. If you ignore this, then you can come up with all kinds of strange results. For example:

$$S = 1 - 1 + 1 - 1 + 1 - 1 + 1 - \cdots$$

Grouping the terms, we get:

$$S = (1 - 1) + (1 - 1) + (1 - 1) + \cdots$$

$$S = 0 + 0 + 0 + 0 + \cdots$$

$$S = 0$$

However, we could also group the series differently:

$$S = 1 - 1 + 1 - 1 + 1 - 1 + 1 - \cdots$$

$$S = 1 + (1 - 1) + (1 - 1) + (1 - 1) + \cdots$$

$$S = 1 + 0 + 0 + 0 + 0 + \cdots$$

$$S = 1$$

Since S is equal to both 1 and 0,

then $1 = 0$.

Here's another example:

$$S = 1 + 2 + 4 + 8 + 16 + 32 + \cdots$$

Subtract 1 from both sides of the equation:

$$S - 1 = 2 + 4 + 8 + 16 + 32 + \cdots$$

Now, we multiply the original equation by 2, we get,

$$2S = 2 + 4 + 8 + 16 + 32 + \cdots$$

Both the second and third equations have the same sum, so,

$$2S = S - 1$$

$$S = -1$$

But it is absurd that the sum of this series could be negative, since all of the terms are positive. The fallacy lies in assuming that a divergent series has a sum. ■

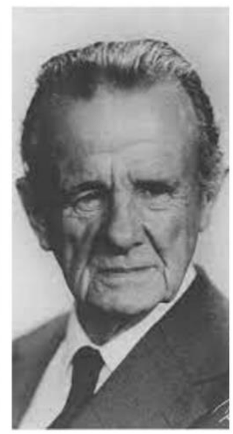
Mathematicians born in September

Adrien-Marie Legendre, was born on 18th September 1752, worked mostly on elliptic integrals provided basic analytical tools for mathematical physics. He is well-known for concepts such as Legendre polynomials and Legendre transformation. The Legendre transformation is commonly used in classical mechanics and in thermodynamics. Associated Legendre polynomials are the most general solution to Legendre's Equation. This is an ordinary differential equation frequently found in physics.

Legendre is also known for his simple proof that π is irrational as well as the first proof that π^2 is irrational.

Harald Cramér, was born on 25th September 1873, a Swedish mathematician, actuary and statistician, specializing in mathematical statistics and probabilistic number theory. He was sometimes referred as *"one of the giants of statistical theory"*. When he first got interested in probability, this was not an accepted branch of mathematics. At that time he realized that there should be a radical change in this field, so he became focused on the rigorous mathematical formulation of probability.

Adrien-Marie Legendre



Harald Cramér

He stated that *"The probability concept should be introduced by a purely mathematical definition, from which its fundamental properties and the classical theorems are deduced by purely mathematical operations."*. ■

Activities

Lecture on Career Opportunities in Mathematics

The Lecture on "Career Opportunities in Mathematics" was organized by Mathematics Club of Department of Mathematics on September 2, 2022. The lecture was delivered by Dr. Jaimikaben Rameshchandra Surawala, Adhyapak Sahayak, Department of Mathematics. Head of the Department Dr. K. J. Chauhan greeted Dr. Surawala and all the present students. Dr. Surawala initiated her talk with the importance of the field of Mathematics. She also discussed the various career options and job opportunities available after undergraduate studies in Mathematics. In addition to this, she explained about the different entrance examinations schemes required for seeking admission in the Central/State Universities, NITs and IITs for Master as well as Ph.D. programme.

Subsequently, she also discussed the various career options and job opportunities available after Post Graduate studies in Mathematics. Finally, she highlighted the numerous research thrust areas of Mathematics for pursuing Ph.D. in Mathematics.

At the end of the session, Dr. Janki Vashi offered vote of thanks and congratulated Dr. Surawala for her wonderful talk. ■

Prof. A. R. Rao Mathematics competition-2022:

To enhance mathematical thinking, understanding and awareness among college students, Prof. A. R. Rao Foundation of Gujarat Ganit Mandal organizes the Prof. A. R. Rao Mathematics Competition for the students of various colleges of Gujarat State. This year competition was held on September 4, 2022 for Third Year B. Sc. and October 2, 2022 for First and Second Year B. Sc. students. In this competition, 36 students from Third Year B. Sc. and 70 students were registered. The following students have achieved success in this competition. It is a matter of pride for department as well as our college.

Result of Prof. A. R. Rao Mathematics Competition:

<u>Third Year B. Sc.:</u>

Second Prize: (2nd Rank)

Mr. Harsh J. Bundeliya (Sem-V)

Appreciation Certificate:

Ms. Drasti S. Bharthaniya (Sem-V)
Ms. Kesar N. Hathiwala (Sem-V)
Ms. Kaushalya D. Ropiya (Sem-V)
Ms. Jogeswari N. Panda (Sem-V)

Ms. Rahi P. Patel (Sem-V)
Mr. Rahul G. Paikaray (Sem-V)
Ms. Mili R. Rathod (Sem-V)
Mr. Roshankumar V. Sah (Sem-V)

First/Second Year B. Sc.:

Second Prize: (2nd Rank)

Mr. Dungarsinh P. Rajput (Sem-I)
Mr. Niket V. Vaghela (Sem-I)

Appreciation Certificate:

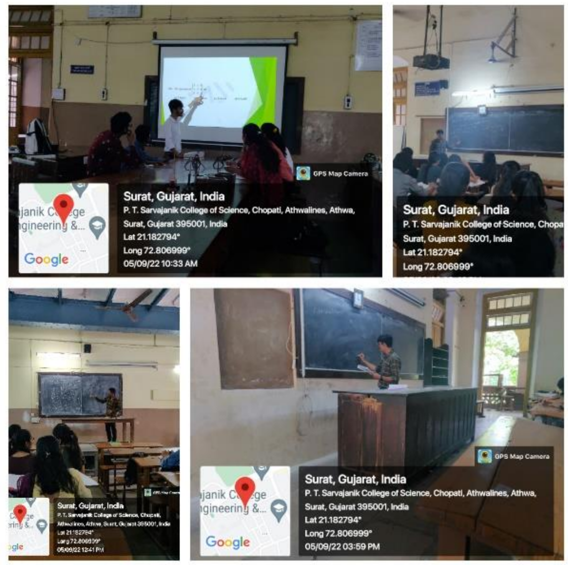
Mr. Devansh Bhutwala (Sem-I)
Mr. Chandresh A. Nagla (Sem-III)

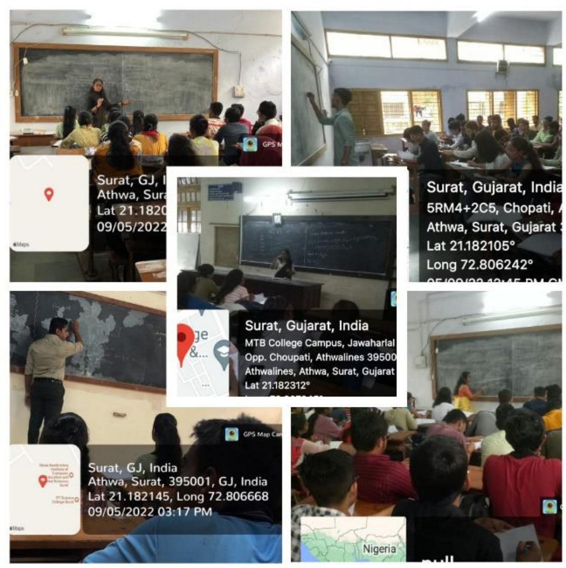
Our Principal Dr. Pruthul R. Desai Sir congratulated all the students for their achievement. The Department of Mathematics also congratulated each students. ■

Celebration of Teacher's Day:

Like every year, this year also the Department of Mathematics celebrated the Teacher's Day on September 5, 2022 with great enthusiasm. All the students were motivated to take part in the event. This year 10 students participated voluntarily and prepared various topics of Mathematics under the able guidance of Dr. K. J. Chauhan, Dr. Janki J. Vashi and Dr. Jaimikaben R. Surawala in advance.

On this day, students performed the role of a teacher, they went to the all classes of B. Sc. Sem-I, Sem-III, Sem-V and taught various topics. In a way they showed their ability to teach and conduct a class confidently. They carried out this activity with skill and devotion. ■





Problem

Why do we use the discriminant $\Delta = b^2 - 4ac$ to solve the quadratic equation $ax^2 + bx + c = 0$. ■